

Stabilization of Unstable Periodic and Aperiodic Orbits of Chaotic Systems by Self-Controlling Feedback

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Naturforsch. **48a**, 629–632 (1993); received February 16, 1993

The methods of stabilization of unstable periodic and aperiodic orbits of a strange attractor with the help of a *small time-continuous perturbation* are discussed. The perturbation is applied to the system in such a way that the desired periodic or aperiodic orbits remain unperturbed. An experimental application of the methods can be carried out by a purely analogous technique without use of any computer.

Almost all investigations of dynamic chaos carried out until recently could be arbitrarily subdivided into three classes: (i) searching for experimental dynamic systems and simple nonlinear theoretical models demonstrating the chaotic behaviour [1]; (ii) analysis of routes to chaos in various dynamic systems [2] with the resulting development of bifurcation theory; (iii) analysis of chaotic states of dynamic systems [3], as a result of which the theory of chaotic time series analysis has been developed. However, some investigations of the last two years can be classified as new emerging subfield in nonlinear science: controlling chaos. The investigations have been initiated by Hübner and Lüscher [4], and later on in a rather different form by Ott, Grebogi, and Yorke [5] (OGY). To convert the chaotic behaviour into periodic time dependence, OGY proposed to stabilize unstable periodic orbits (UPO) which are dense in a typical strange attractor. This can be achieved by feeding a *small* time-dependent perturbation to the system in the form of a feedback.

The OGY method is discrete in time as it deals with the Poincaré map of the system. In experiment it requires a permanent computer analysis of the state of the system. Here we present three methods of the time-continuous control, available for experimental applications by a purely analogous technique. We extend the ideas of [6] for stabilization of aperiodic orbits (AO) of the strange attractor.

Let us consider a chaotic dynamic system which can be simulated by ordinary differential equations:

$$\frac{dy}{dt} = P(y, \mathbf{x}) + F(t), \quad \frac{d\mathbf{x}}{dt} = Q(y, \mathbf{x}). \quad (1)$$

We imagine that the equations (1) are unknown, but some scalar variable $y(t)$ can be measured as a system output. Here vector $\mathbf{x}(t)$ describes the remaining variables of the system which are not available or not of interest for observation and $F(t)$ is an external perturbation corresponding to the system input. It has been demonstrated, using a standard method of delay-coordinates, that a large number of distinct UPO on a chaotic attractor can be obtained from one scalar signal [7]. Applying this method to our system, we can determine from the experimentally measured output signal $y(t)$ various periodic signals of different form $y = y_i(t)$, $y_i(t + T_i) = y_i(t)$ corresponding to different UPO's. Here T_i is the period of the i -th UPO. Then we examine these periodic signals and select the one which we intend to stabilize. To achieve this goal we have to design a special external oscillator which generates a signal proportional to $y_i(t)$. The difference between signal $y_i(t)$ and output signal $y(t)$ is used as a control signal:

$$F(t) = K \{y_i(t) - y(t)\}. \quad (2)$$

Here K is an experimentally adjustable weight of the perturbation. The perturbation has to be introduced to the system input as a negative feedback ($K > 0$). The important feature of the perturbation (2) is that it does not change the solution of (1) corresponding to the i -th UPO. By selecting the weight K , one can achieve its stabilization. When stabilization is achieved, the output signal $y(t)$ is very close to $y_i(t)$ and the

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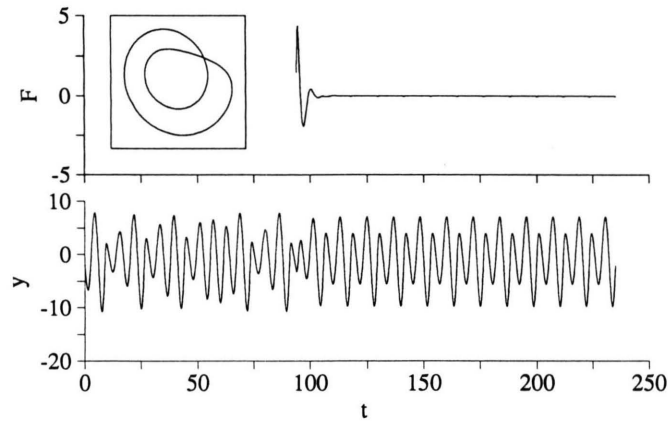


Fig. 1. Dynamics of the output signal $y(t)$ and the perturbation $F(t)$ for the Rössler system: $dx/dt = -y - z$, $dy/dt = x + 0.2y + F(t)$, $dz/dt = 0.2 + z(x - 5.7)$. The perturbation $F(t)$ is determined by (2), $K = 0.4$, $y_i(t)$ corresponds to the period-2 cycle. The origin of the curve F corresponds to the moment of switching on the perturbation. The inset shows the x - y phase portrait of the system in the post-transient regime. The initial conditions for the UPO y_i and the current state y of the Rössler system are $\{-2.250, -1.734, -1.235\}$ and $\{-0.459, -3.210, 0.030\}$, respectively.

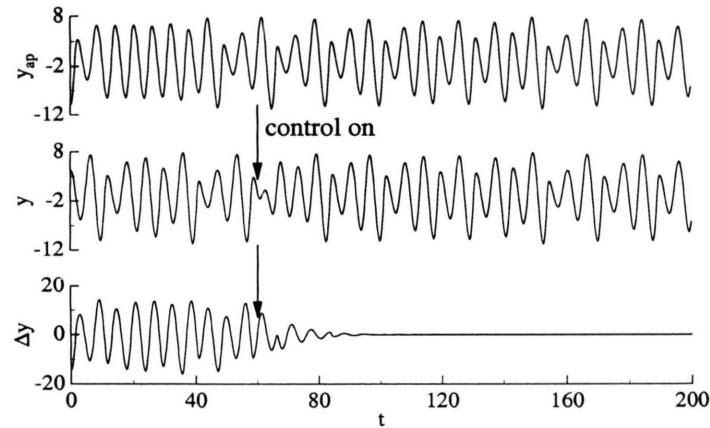


Fig. 3. Segment of a "recorded" aperiodic output signal $y_{ap}(t)$ of an unperturbed Rössler system and the dynamics of the output signal $y(t)$ of the perturbed system and the difference $\Delta y(t) = y_{ap} - y$. The arrows show the moment of switching on the perturbation, $K = 0.4$. The initial conditions of the Rössler system for the "recorded" AO y_{ap} and the current state y are $\{1.022, -9.968, 0.033\}$ and $\{-0.151, 4.169, 0.041\}$, respectively.

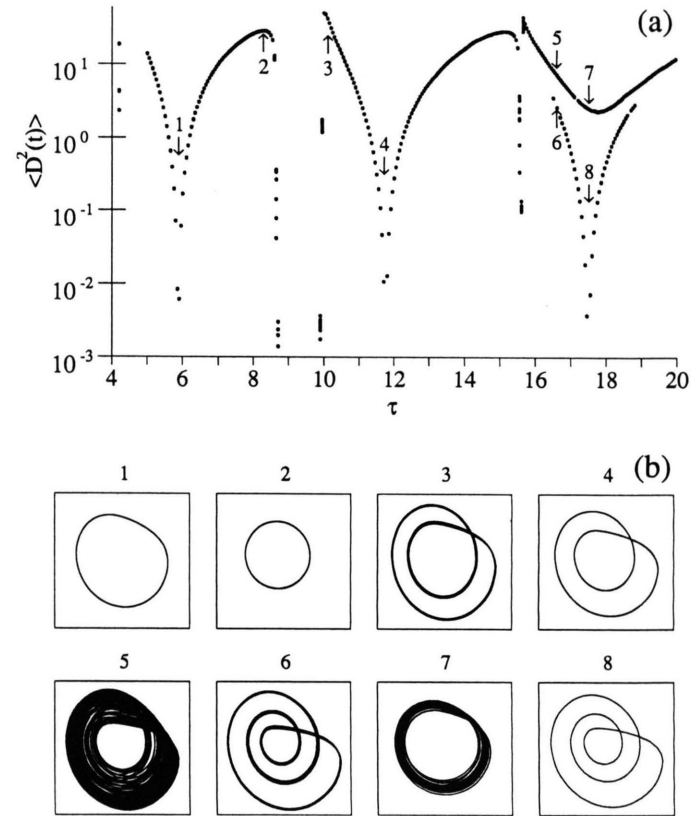


Fig. 2. (a) Dependence of the dispersion $\langle D^2(t) \rangle$ of the perturbation (3) on the delay time τ and (b) the x - y phase portraits of the Rössler system at some values of delay time in the case of delayed feedback control. For each value of τ , 20 values of the dispersion, corresponding to 20 different initial conditions, are depicted. The minima of the resonance curves (1, 4, 8) are located at the points where the delay time coincides with the periods of the UPO: $\tau = T_i$. They correspond to initially unstable period-1, -2, and -3 cycles. The resonance curves are separated by additional minima intervals, corresponding to the stabilization of the unstable fixed point of the Rössler system. For large τ the system has two stable solutions depending on the initial conditions. The phase portraits 5 and 6 as well as 7 and 8 have been obtained for the same values of the delay time, but with different initial conditions, $K = 0.2$.

perturbation, as in the OGY method, is very small. Figure 1 shows the results of such stabilization for the Rössler system. The values of K leading to the stabilization are determined by the sign of the maximal Lyapunov exponent of the perturbed system. Analysis of this exponent shows that the period-1 and -2 UPO's of the Rössler system can be stabilized in the intervals of the parameter $K = [0.19, 7.9]$ and $K = [0.22, \infty]$, respectively. The experimental application of this method can be divided into two stages. In the first, preparatory stage the output signal should be investigated and the oscillator generating a periodical signal proportional to $y_i(t)$ should be designed. In the second stage the control is achieved simply by combining a self-controlling circuit, which uses the difference between an output signal and the signal of the external oscillator.

The complexity of the experimental realization of the above method lies mainly in the design of a special periodic oscillator. The second method which we have considered has no such shortcoming. The idea of this method consists of substituting the external signal $y_i(t)$ in (2) for the delayed output signal $y(t - \tau)$:

$$F(t) = K \{y(t - \tau) - y(t)\} \equiv KD(t). \quad (3)$$

Here τ is a delay time. This perturbation as well as in the form (2) vanishes for the i -th UPO at the delay time coinciding with their period: $\tau = T_i$. Choosing an appropriate weight K of the feedback, one can achieve the stabilization. The results are very similar to those presented in Fig. 1, however an experimental realization is simpler in this case. No external perturbation or computer, but only a simple delay line is needed for this control. The control is achieved by an output signal which is fed in a special form to the input. The difference between the delayed output signal and the output signal itself is used as a control signal. This feedback performs the self-control. To achieve the stabilization of the desired UPO, two parameters, namely the time of delay τ and the weight K of the feedback should be adjusted in the experiment. The amplitude of the feedback signal can be considered as a criterion of UPO's stabilization. When the system moves along its UPO, this amplitude is extremely small. The dependence of this amplitude on the delay time for the Rössler system is illustrated in Figure 2. The analysis of the maximal Lyapunov exponent in

this case shows that the period-1 and period-2 UPO's can be stabilized in the intervals $[0.12, 0.65]$ and $[0.1, 0.3]$ of the parameter K , respectively. These intervals are much narrower than those of the external force control. This means that the delayed feedback control is more sensitive to the fitting of the parameters.

It is possible to extend the first method for stabilization of an AO on the strange attractor; to do this, the periodic signal $y_i(t)$ in (2) should be substituted for an arbitrarily chosen segment of the aperiodic output signal $y_{ap}(t)$ of the unperturbed system:

$$F(t) = K \{y_{ap}(t) - y(t)\}. \quad (4)$$

The signal $y_{ap}(t)$ represents the y -component of the AO corresponding to the asymptotic solution of the unperturbed system (1). The perturbation (4) can stabilize this AO, leaving it unperturbed. Figure 3 illustrates the results of such a stabilization for one specific segment of the AO on the Rössler strange attractor. The distance between the current phase point of the perturbed system and the chosen AO changes exponentially. This permits the use of the conditional Lyapunov exponent [8] of the perturbed system as a criterion of the stabilization. The perturbation (4) inverts the sign of the maximal conditional Lyapunov exponent of the Rössler system in the interval of the parameter $K = [0.12, \infty]$. Therefore, in this interval of K an unpredictable chaos turns into a predictable one with the help of only a *small* perturbation. An experimental realization of this method is very simple. An appropriate segment of the output signal of the unperturbed system has to be singled out and recorded in a memory. With the help of only a *small feedback perturbation*, using the difference between the output and the recorded signal, the system can be forced to repeat exactly the recorded signal.

The methods presented have been tested for many chaotic models. They work effectively even at the presence of a rather large noise.

The author acknowledges R. P. Hübener, A. Kittel, J. Parisi, J. Peinke, R. Richter, and O. E. Rössler for discussions and the Alexander von Humboldt Foundation for financial support. The paper has been presented at the 3rd Annual Meeting of ENGADYN, Grenoble, 1992.

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